

NAG Toolbox for MATLAB

f04at

1 Purpose

f04at calculates the accurate solution of a set of real linear equations with a single right-hand side, using an LU factorization with partial pivoting, and iterative refinement.

2 Syntax

```
[c, aa, ifail] = f04at(a, b, 'n', n)
```

3 Description

Given a set of real linear equations, $Ax = b$, the function first computes an LU factorization of A with partial pivoting, $PA = LU$, where P is a permutation matrix, L is lower triangular and U is unit upper triangular. An approximation to x is found by forward and backward substitution in $Ly = Pb$ and $Ux = y$. The residual vector $r = b - Ax$ is then calculated using **additional precision**, and a correction d to x is found by solving $LUd = r$. x is replaced by $x + d$, and this iterative refinement of the solution is repeated until full machine accuracy is obtained.

4 References

Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer-Verlag

5 Parameters

5.1 Compulsory Input Parameters

- 1: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n matrix A .

- 2: **b(*)** – double array

Note: the dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The right-hand side vector b .

5.2 Optional Input Parameters

- 1: **n** – int32 scalar

Default: The second dimension of the array **a** The dimension of the array **b**.
 n , the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldaa, wks1, wks2

5.4 Output Parameters

1: **c(*)** – double array

Note: the dimension of the array **c** must be at least $\max(1, \mathbf{n})$.

The solution vector x .

2: **aa(ldaa,*)** – double array

The first dimension of the array **aa** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The triangular factors L and U , except that the unit diagonal elements of U are not stored.

3: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

The matrix A is singular, possibly due to rounding errors.

ifail = 2

Iterative refinement fails to improve the solution, i.e., the matrix A is too ill-conditioned.

ifail = 3

On entry, $\mathbf{n} < 0$,
or $\mathbf{lda} < \max(1, \mathbf{n})$,
or $\mathbf{ldaa} < \max(1, \mathbf{n})$.

7 Accuracy

The computed solutions should be correct to full machine accuracy. For a detailed error analysis see page 107 of Wilkinson and Reinsch 1971.

8 Further Comments

The time taken by f04at is approximately proportional to n^3 .

The function **must not** be called with the same name for parameters **b** and **c**.

9 Example

```
a = [33, 16, 72;  
     -24, -10, -57;  
     -8, -4, -17];  
b = [-359;  
     281;  
     85];  
[c, aa, ifail] = f04at(a, b)  
  
c =  
     1  
    -2
```

```
      -5
aa =
-8.0000    0.5000    2.1250
-24.0000    2.0000   -3.0000
 33.0000   -0.5000    0.3750
ifail =
      0
```
